

# Package: candisc (via r-universe)

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**Maintainer** Michael Friendly <friendly@yorku.ca>

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**Description** Functions for computing and visualizing generalized canonical discriminant analyses and canonical correlation analysis for a multivariate linear model. Traditional canonical discriminant analysis is restricted to a one-way 'MANOVA' design and is equivalent to canonical correlation analysis between a set of quantitative response variables and a set of dummy variables coded from the factor variable. The 'candisc' package generalizes this to higher-way 'MANOVA' designs for all factors in a multivariate linear model, computing canonical scores and vectors for each term. The graphic functions provide low-rank (1D, 2D, 3D) visualizations of terms in an 'mlm' via the 'plot.candisc' and 'heplot.candisc' methods. Related plots are now provided for canonical correlation analysis when all predictors are quantitative.

**License** GPL ( $\geq 2$ )

**Encoding** UTF-8

**VignetteBuilder** knitr

**URL** <https://github.com/friendly/candisc/>,  
<https://friendly.github.io/candisc/>

**BugReports** <https://github.com/friendly/candisc/issues>

**RoxygenNote** 7.3.1

**Repository** <https://friendly.r-universe.dev>

**RemoteUrl** <https://github.com/friendly/candisc>

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## Contents

candisc-package . . . . .	2
cancor . . . . .	4
candisc . . . . .	10
candiscList . . . . .	15
can_lm . . . . .	16
dataIndex . . . . .	17
Grass . . . . .	18
heplot.cancor . . . . .	19
heplot.candisc . . . . .	22
heplot.candiscList . . . . .	25
HSB . . . . .	26
plot.cancor . . . . .	27
predictor.names . . . . .	30
redundancy . . . . .	31
varOrder . . . . .	32
vecscale . . . . .	35
vectors . . . . .	36
Wilks . . . . .	37
Wine . . . . .	38
Wolves . . . . .	40
<b>Index</b>	<b>42</b>

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candisc-package	<i>Visualizing Generalized Canonical Discriminant and Canonical Correlation Analysis</i>
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## Description

This package includes functions for computing and visualizing generalized canonical discriminant analyses and canonical correlation analysis for a multivariate linear model. The goal is to provide ways of visualizing such models in a low-dimensional space corresponding to dimensions (linear combinations of the response variables) of maximal relationship to the predictor variables.

## Details

Traditional canonical discriminant analysis is restricted to a one-way MANOVA design and is equivalent to canonical correlation analysis between a set of quantitative response variables and a set of dummy variables coded from the factor variable. The `candisc` package generalizes this to multi-way MANOVA designs for all terms in a multivariate linear model (i.e., an `mlm` object), computing canonical scores and vectors for each term (giving a `candiscList` object).

The graphic functions are designed to provide low-rank (1D, 2D, 3D) visualizations of terms in a `mlm` via the `plot.candisc` method, and the HE plot `heplot.candisc` and `heplot3d.candisc` methods. For `mlms` with more than a few response variables, these methods often provide a much simpler interpretation of the nature of effects in canonical space than `heplots` for pairs of responses or an HE plot matrix of all responses in variable space.

Analogously, a multivariate linear (regression) model with quantitative predictors can also be represented in a reduced-rank space by means of a canonical correlation transformation of the Y and X variables to uncorrelated canonical variates, `Ycan` and `Xcan`. Computation for this analysis is provided by `cancor` and related methods. Visualization of these results in canonical space are provided by the `plot.cancor`, `heplot.cancor` and `heplot3d.cancor` methods.

These relations among response variables in linear models can also be useful for “effect ordering” (Friendly & Kwan (2003) for *variables* in other multivariate data displays to make the displayed relationships more coherent. The function `varOrder` implements a collection of these methods.

A new vignette, `vignette("diabetes", package="candisc")`, illustrates some of these methods. A more comprehensive collection of examples is contained in the vignette for the `heplots` package, `vignette("HE-examples", package="heplots")`.

The organization of functions in this package and the `heplots` package may change in a later version.

## Author(s)

Michael Friendly and John Fox

Maintainer: Michael Friendly <friendly@yorku.ca>

## References

- Friendly, M. (2007). HE plots for Multivariate General Linear Models. *Journal of Computational and Graphical Statistics*, **16**(2) 421–444. <http://datavis.ca/papers/jcgs-heplots.pdf>, doi:10.1198/106186007X208407.
- Friendly, M. & Kwan, E. (2003). Effect Ordering for Data Displays, *Computational Statistics and Data Analysis*, **43**, 509-539. doi:10.1016/S01679473(02)002906
- Friendly, M. & Sigal, M. (2014). Recent Advances in Visualizing Multivariate Linear Models. *Revista Colombiana de Estadística*, **37**(2), 261-283. doi:10.15446/rce.v37n2spe.47934.
- Friendly, M. & Sigal, M. (2017). Graphical Methods for Multivariate Linear Models in Psychological Research: An R Tutorial, *The Quantitative Methods for Psychology*, **13** (1), 20-45. doi:10.20982/tqmp.13.1.p020.
- Gittins, R. (1985). *Canonical Analysis: A Review with Applications in Ecology*, Berlin: Springer.

**See Also**

[heplot](#) for details about HE plots.

[candisc](#), [cancor](#) for details about canonical discriminant analysis and canonical correlation analysis.

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cancor

*Canonical Correlation Analysis*

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**Description**

The function `cancor` generalizes and regularizes computation for canonical correlation analysis in a way conducive to visualization using methods in the [heplots](#) package.

The package provides the following display, extractor and plotting methods for "cancor" objects

`print()`, `summary()` Print and summarise the CCA

`coef()` Extract coefficients for X, Y, or both

`scores()` Extract observation scores on the canonical variables

`redundancy()` Redundancy analysis: proportion of variances of the variables in each set (X and Y) accounted for by the variables in the other set through the canonical variates

`plot()` Plot pairs of canonical scores with a data ellipse and regression line

`heplot()` HE plot of the Y canonical variables showing effects of the X variables and projections of the Y variables in this space.

As well, the function provides for observation weights, which may be useful in some situations, as well as providing a basis for robust methods in which potential outliers can be down-weighted.

**Usage**

```
cancor(x, ...)
```

```
## S3 method for class 'formula'
```

```
cancor(formula, data, subset, weights, na.rm = TRUE, method = "gensvd", ...)
```

```
## Default S3 method:
```

```
cancor(
  x,
  y,
  weights,
  X.names = colnames(x),
  Y.names = colnames(y),
  row.names = rownames(x),
  xcenter = TRUE,
  ycenter = TRUE,
  xscale = FALSE,
  yscale = FALSE,
```

```

    ndim = min(p, q),
    set.names = c("X", "Y"),
    prefix = c("Xcan", "Ycan"),
    na.rm = TRUE,
    use = if (na.rm) "complete" else "pairwise",
    method = "gensvd",
    ...
)

## S3 method for class 'cancel'
print(x, digits = max(getOption("digits") - 2, 3), ...)

## S3 method for class 'cancel'
summary(object, digits = max(getOption("digits") - 2, 3), ...)

## S3 method for class 'cancel'
scores(x, type = c("x", "y", "both", "list", "data.frame"), ...)

## S3 method for class 'cancel'
coef(object, type = c("x", "y", "both", "list"), standardize = FALSE, ...)

```

### Arguments

x	Varies depending on method. For the <code>cancel.default</code> method, this should be a matrix or <code>data.frame</code> whose columns contain the X variables
...	Other arguments, passed to methods
formula	A two-sided formula of the form <code>cbind(y1, y2, y3, ...) ~ x1 + x2 + x3 + ...</code>
data	The <code>data.frame</code> within which the formula is evaluated
subset	an optional vector specifying a subset of observations to be used in the calculations.
weights	Observation weights. If supplied, this must be a vector of length equal to the number of observations in X and Y, typically within [0,1]. In that case, the variance-covariance matrices are computed using <code>cov.wt</code> , and the number of observations is taken as the number of non-zero weights.
na.rm	logical, determining whether observations with missing cases are excluded in the computation of the variance matrix of (X,Y). See Notes for details on missing data.
method	the method to be used for calculation; currently only <code>method = "gensvd"</code> is supported;
y	For the <code>cancel.default</code> method, a matrix or <code>data.frame</code> whose columns contain the Y variables
X.names, Y.names	Character vectors of names for the X and Y variables.
row.names	Observation names in x, y
xcenter, ycenter	logical. Center the X, Y variables? [not yet implemented]

<code>xscale, yscale</code>	logical. Scale the X, Y variables to unit variance? [not yet implemented]
<code>ndim</code>	Number of canonical dimensions to retain in the result, for scores, coefficients, etc.
<code>set.names</code>	A vector of two character strings, giving names for the collections of the X, Y variables.
<code>prefix</code>	A vector of two character strings, giving prefixes used to name the X and Y canonical variables, respectively.
<code>use</code>	argument passed to <code>var</code> determining how missing data are handled. Only the default, <code>use="complete"</code> is allowed when observation weights are supplied.
<code>digits</code>	Number of digits passed to <code>print</code> and <code>summary</code> methods
<code>object</code>	A <code>cancor</code> object for related methods.
<code>type</code>	For the <code>coef</code> method, the type of coefficients returned, one of "x", "y", "both". For the <code>scores</code> method, the same list, or "data.frame", which returns a <code>data.frame</code> containing the X and Y canonical scores.
<code>standardize</code>	For the <code>coef</code> method, whether coefficients should be standardized by dividing by the standard deviations of the X and Y variables.

## Details

Canonical correlation analysis (CCA), as traditionally presented is used to identify and measure the associations between two sets of quantitative variables, X and Y. It is often used in the same situations for which a multivariate multiple regression analysis (MMRA) would be used.

However, CCA is "symmetric" in that the sets X and Y have equivalent status, and the goal is to find orthogonal linear combinations of each having maximal (canonical) correlations. On the other hand, MMRA is "asymmetric", in that the Y set is considered as responses, *each one* to be explained by *separate* linear combinations of the Xs.

Let  $\mathbf{Y}_{n \times p}$  and  $\mathbf{X}_{n \times q}$  be two sets of variables over which CCA is computed. We find canonical coefficients  $\mathbf{A}_{p \times k}$  and  $\mathbf{B}_{q \times k}$ ,  $k = \min(p, q)$  such that the canonical variables

$$\mathbf{U}_{n \times k} = \mathbf{Y}\mathbf{A} \quad \text{and} \quad \mathbf{V}_{n \times k} = \mathbf{X}\mathbf{B}$$

have maximal, diagonal correlation structure. That is, the coefficients  $\mathbf{A}$  and  $\mathbf{B}$  are chosen such that the (canonical) correlations between each pair  $r_i = \text{cor}(\mathbf{u}_i, \mathbf{v}_i)$ ,  $i = 1, 2, \dots, k$  are maximized and all other pairs are uncorrelated,  $r_{ij} = \text{cor}(\mathbf{u}_i, \mathbf{v}_j) = 0$ ,  $i \neq j$ . Thus, all correlations between the X and Y variables are channeled through the correlations between the pairs of canonical variates.

For visualization using HE plots, it is most natural to consider plots representing the relations among the canonical variables for the Y variables in terms of a multivariate linear model predicting the Y canonical scores, using either the X variables or the X canonical scores as predictors. Such plots, using `heplot.cancor` provide a low-rank (1D, 2D, 3D) visualization of the relations between the two sets, and so are useful in cases when there are more than 2 or 3 variables in each of X and Y.

The connection between CCA and HE plots for MMRA models can be developed as follows. CCA can also be viewed as a principal component transformation of the predicted values of one set of variables from a regression on the other set of variables, in the metric of the error covariance matrix.

For example, regress the Y variables on the X variables, giving predicted values  $\hat{Y} = X(X'X)^{-1}X'Y$  and residuals  $R = Y - \hat{Y}$ . The error covariance matrix is  $E = R'R/(n - 1)$ . Choose a transformation Q that orthogonalizes the error covariance matrix to an identity, that is,  $(RQ)'(RQ) =$

$Q'R'RQ = (n - 1)I$ , and apply the same transformation to the predicted values to yield, say,  $Z = \hat{Y}Q$ . Then, a principal component analysis on the covariance matrix of  $Z$  gives eigenvalues of  $E^{-1}H$ , and so is equivalent to the MMRA analysis of  $\text{lm}(Y \sim X)$  statistically, but visualized here in canonical space.

### Value

An object of class `cancorr`, a list with the following components:

<code>cancor</code>	Canonical correlations, i.e., the correlations between each canonical variate for the Y variables with the corresponding canonical variate for the X variables.
<code>names</code>	Names for various items, a list of 4 components: X, Y, row.names, set.names
<code>ndim</code>	Number of canonical dimensions extracted, $\leq \min(p, q)$
<code>dim</code>	Problem dimensions, a list of 3 components: p (number of X variables), q (number of Y variables), n (sample size)
<code>coef</code>	Canonical coefficients, a list of 2 components: X, Y
<code>scores</code>	Canonical variate scores, a list of 2 components: X, Y
<code>scores</code>	Canonical variate scores, a list of 2 components: X Canonical variate scores for the X variables Y Canonical variate scores for the Y variables
<code>X</code>	The matrix X
<code>Y</code>	The matrix Y
<code>weights</code>	Observation weights, if supplied, else NULL
<code>structure</code>	Structure correlations, a list of 4 components: X.xscores, Y.xscores, X.yscores, Y.yscores
<code>structure</code>	Structure correlations ("loadings"), a list of 4 components: <b>X.xscores</b> Structure correlations of the X variables with the Xcan canonical scores <b>Y.xscores</b> Structure correlations of the Y variables with the Xcan canonical scores <b>X.yscores</b> Structure correlations of the X variables with the Ycan canonical scores <b>Y.yscores</b> Structure correlations of the Y variables with the Ycan canonical scores

The formula method also returns components `call` and `terms`

### Methods (by class)

- `cancor(formula)`: "formula" method.
- `cancor(default)`: "default" method.

**Methods (by generic)**

- `print(cancor)`: `print()` method for "cancor" objects.
- `summary(cancor)`: `summary()` method for "cancor" objects.
- `scores(cancor)`: `scores()` method for "cancor" objects.
- `coef(cancor)`: `coef()` method for "cancor" objects.

**Note**

Not all features of CCA are presently implemented: standardized vs. raw scores, more flexible handling of missing data, other plot methods, ...

**Author(s)**

Michael Friendly

**References**

- Gittins, R. (1985). *Canonical Analysis: A Review with Applications in Ecology*, Berlin: Springer.
- Mardia, K. V., Kent, J. T. and Bibby, J. M. (1979). *Multivariate Analysis*. London: Academic Press.

**See Also**

Other implementations of CCA: [cancor](#) (very basic), [cca](#) in the **yacca** (fairly complete, but very messy return structure), [cc](#) in **CCA** (fairly complete, very messy return structure, no longer maintained).

[redundancy](#), for redundancy analysis; [plot.cancor](#), for enhanced scatterplots of the canonical variates.

[heplot.cancor](#) for CCA HE plots and [heplots](#) for generic heplot methods.

[candisc](#) for related methods focused on multivariate linear models with one or more factors among the X variables.

**Examples**

```
data(Rohwer, package="heplots")
X <- as.matrix(Rohwer[,6:10]) # the PA tests
Y <- as.matrix(Rohwer[,3:5]) # the aptitude/ability variables

# visualize the correlation matrix using corrplot()
if (require(corrplot)) {
  M <- cor(cbind(X,Y))
  corrplot(M, method="ellipse", order="hclust", addrect=2, addCoef.col="black")
}

(cc <- cancor(X, Y, set.names=c("PA", "Ability"))))

## Canonical correlation analysis of:
##      5 PA variables: n, s, ns, na, ss
```



```

## with      3  Ability variables: SAT, PPVT, Raven
##
##      CanR  CanRSQ  Eigen percent      cum              scree
## 1 0.6703 0.44934 0.81599  77.30  77.30 *****
## 2 0.3837 0.14719 0.17260  16.35  93.65 *****
## 3 0.2506 0.06282 0.06704   6.35 100.00 **
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
##
##      CanR  WilksL      F df1  df2 p.value
## 1 0.67033 0.44011 3.8961 15 168.8 0.000006
## 2 0.38366 0.79923 1.8379  8 124.0 0.076076
## 3 0.25065 0.93718 1.4078  3  63.0 0.248814

# formula method
cc <- cancort(cbind(SAT, PPVT, Raven) ~ n + s + ns + na + ss, data=Rohwer,
  set.names=c("PA", "Ability"))

# using observation weights
set.seed(12345)
wts <- sample(0:1, size=nrow(Rohwer), replace=TRUE, prob=c(.05, .95))
(ccw <- cancort(X, Y, set.names=c("PA", "Ability"), weights=wts) )

# show correlations of the canonical scores
zapsmall(cor(scores(cc, type="x"), scores(cc, type="y")))

# standardized coefficients
coef(cc, type="both", standardize=TRUE)

# plot canonical scores
plot(cc,
  smooth=TRUE, pch=16, id.n = 3)
text(-2, 1.5, paste("Can R =", round(cc$cancor[1], 3)), pos = 4)
plot(cc, which = 2,
  smooth=TRUE, pch=16, id.n = 3)
text(-2.2, 2.5, paste("Can R =", round(cc$cancor[2], 3)), pos = 4)

#####
data(schooldata)
#####

#fit the MMreg model
school.mod <- lm(cbind(reading, mathematics, selfesteem) ~
  education + occupation + visit + counseling + teacher, data=schooldata)
car::Anova(school.mod)
pairs(school.mod)

# canonical correlation analysis
school.cc <- cancort(cbind(reading, mathematics, selfesteem) ~
  education + occupation + visit + counseling + teacher, data=schooldata)
school.cc

```

```
heplot(school.cc, xpd=TRUE, scale=0.3)
```

---

candisc

*Canonical discriminant analysis*


---

### Description

candisc performs a generalized canonical discriminant analysis for one term in a multivariate linear model (i.e., an `mlm` object), computing canonical scores and vectors. It represents a transformation of the original variables into a canonical space of maximal differences for the term, controlling for other model terms.

### Usage

```
candisc(mod, ...)

## S3 method for class 'mlm'
candisc(mod, term, type = "2", manova, ndim = rank, ...)

## S3 method for class 'candisc'
print(x, digits = max(getOption("digits") - 2, 3), LRtests = TRUE, ...)

## S3 method for class 'candisc'
summary(
  object,
  means = TRUE,
  scores = FALSE,
  coef = c("std"),
  ndim,
  digits = max(getOption("digits") - 2, 4),
  ...
)

## S3 method for class 'candisc'
coef(object, type = c("std", "raw", "structure"), ...)

## S3 method for class 'candisc'
plot(
  x,
  which = 1:2,
  conf = 0.95,
  col,
  pch,
  scale,
  asp = 1,
```

```

var.col = "blue",
var.lwd = par("lwd"),
var.labels,
var.cex = 1,
var.pos,
rev.axes = c(FALSE, FALSE),
ellipse = FALSE,
ellipse.prob = 0.68,
fill.alpha = 0.1,
prefix = "Can",
suffix = TRUE,
titles.1d = c("Canonical scores", "Structure"),
points.1d = FALSE,
...
)

```

### Arguments

mod	An <code>mlm</code> object, such as computed by <code>lm()</code> with a multivariate response
...	arguments to be passed down. In particular, <code>type="n"</code> can be used with the <code>plot</code> method to suppress the display of canonical scores.
term	the name of one term from <code>mod</code> for which the canonical analysis is performed.
type	type of test for the model term, one of: "II", "III", "2", or "3"
manova	the <code>Anova.mlm</code> object corresponding to <code>mod</code> . Normally, this is computed internally by <code>Anova(mod)</code>
ndim	Number of dimensions to store in (or retrieve from, for the summary method) the means, structure, scores and <code>coeffs.*</code> components. The default is the rank of the H matrix for the hypothesis term.
digits	significant digits to print.
LRtests	logical; should likelihood ratio tests for the canonical dimensions be printed?
object, x	A <code>candisc</code> object
means	Logical value used to determine if canonical means are printed
scores	Logical value used to determine if canonical scores are printed
coef	Type of coefficients printed by the summary method. Any one or more of "std", "raw", or "structure"
which	A vector of one or two integers, selecting the canonical dimension(s) to plot. If the canonical structure for a term has <code>ndim==1</code> , or <code>length(which)==1</code> , a 1D representation of canonical scores and structure coefficients is produced by the <code>plot</code> method. Otherwise, a 2D plot is produced.
conf	Confidence coefficient for the confidence circles around canonical means plotted in the <code>plot</code> method
col	A vector of the unique colors to be used for the levels of the term in the <code>plot</code> method, one for each level of the term. In this version, you should assign colors and point symbols explicitly, rather than relying on the somewhat arbitrary defaults, based on <a href="#">palette</a>

<code>pch</code>	A vector of the unique point symbols to be used for the levels of the term in the plot method
<code>scale</code>	Scale factor for the variable vectors in canonical space. If not specified, a scale factor is calculated to make the variable vectors approximately fill the plot space.
<code>asp</code>	Aspect ratio for the plot method. The <code>asp=1</code> (the default) assures that the units on the horizontal and vertical axes are the same, so that lengths and angles of the variable vectors are interpretable.
<code>var.col</code>	Color used to plot variable vectors
<code>var.lwd</code>	Line width used to plot variable vectors
<code>var.labels</code>	Optional vector of variable labels to replace variable names in the plots
<code>var.cex</code>	Character expansion size for variable labels in the plots
<code>var.pos</code>	Position(s) of variable vector labels wrt. the end point. If not specified, the labels are out-justified left and right with respect to the end points.
<code>rev.axes</code>	Logical, a vector of length(which). TRUE causes the orientation of the canonical scores and structure coefficients to be reversed along a given axis.
<code>ellipse</code>	Draw data ellipses for canonical scores?
<code>ellipse.prob</code>	Coverage probability for the data ellipses
<code>fill.alpha</code>	Transparency value for the color used to fill the ellipses. Use <code>fill.alpha</code> to draw the ellipses unfilled.
<code>prefix</code>	Prefix used to label the canonical dimensions plotted
<code>suffix</code>	Suffix for labels of canonical dimensions. If <code>suffix=TRUE</code> the percent of hypothesis (H) variance accounted for by each canonical dimension is added to the axis label.
<code>titles.1d</code>	A character vector of length 2, containing titles for the panels used to plot the canonical scores and structure vectors, for the case in which there is only one canonical dimension.
<code>points.1d</code>	Logical value for <code>plot.candisc</code> when only one canonical dimension.

## Details

In typical usage, the term should be a factor or interaction corresponding to a multivariate test with 2 or more degrees of freedom for the null hypothesis.

Canonical discriminant analysis is typically carried out in conjunction with a one-way MANOVA design. It represents a linear transformation of the response variables into a canonical space in which (a) each successive canonical variate produces maximal separation among the groups (e.g., maximum univariate F statistics), and (b) all canonical variates are mutually uncorrelated. For a one-way MANOVA with  $g$  groups and  $p$  responses, there are  $dfh = \min(g-1, p)$  such canonical dimensions, and tests, initially stated by Bartlett (1938) allow one to determine the number of significant canonical dimensions.

Computational details for the one-way case are described in Cooley & Lohnes (1971), and in the *SAS/STAT User's Guide*, "The CANDISC procedure: Computational Details," [http://support.sas.com/documentation/cdl/en/statug/63962/HTML/default/viewer.htm#statug\\_candisc\\_sect012.htm](http://support.sas.com/documentation/cdl/en/statug/63962/HTML/default/viewer.htm#statug_candisc_sect012.htm).

A generalized canonical discriminant analysis extends this idea to a general multivariate linear model. Analysis of each term in the `m1m` produces a rank  $df_h$  H matrix sum of squares and crossproducts matrix that is tested against the rank  $df_e$  E matrix by the standard multivariate tests (Wilks' Lambda, Hotelling-Lawley trace, Pillai trace, Roy's maximum root test). For any given term in the `m1m`, the generalized canonical discriminant analysis amounts to a standard discriminant analysis based on the H matrix for that term in relation to the full-model E matrix.

The plot method for `candisc` objects is typically a 2D plot, similar to a biplot. It shows the canonical scores for the groups defined by the term as points and the canonical structure coefficients as vectors from the origin.

If the canonical structure for a term has `ndim==1`, or `length(which)==1`, the 1D representation consists of a boxplot of canonical scores and a vector diagram showing the magnitudes of the structure coefficients.

## Value

An object of class `candisc` with the following components:

<code>dfh</code>	hypothesis degrees of freedom for term
<code>dfe</code>	error degrees of freedom for the <code>m1m</code>
<code>rank</code>	number of non-zero eigenvalues of $HE^{-1}$
<code>eigenvalues</code>	eigenvalues of $HE^{-1}$
<code>canrsq</code>	squared canonical correlations
<code>pct</code>	A vector containing the percentages of the <code>canrsq</code> of their total.
<code>ndim</code>	Number of canonical dimensions stored in the <code>means</code> , <code>structure</code> and <code>coeffs.*</code> components
<code>means</code>	A <code>data.frame</code> containing the class means for the levels of the factor(s) in the term
<code>factors</code>	A data frame containing the levels of the factor(s) in the term
<code>term</code>	name of the term
<code>terms</code>	A character vector containing the names of the terms in the <code>m1m</code> object
<code>coeffs.raw</code>	A matrix containing the raw canonical coefficients
<code>coeffs.std</code>	A matrix containing the standardized canonical coefficients
<code>structure</code>	A matrix containing the canonical structure coefficients on <code>ndim</code> dimensions, i.e., the correlations between the original variates and the canonical scores. These are sometimes referred to as Total Structure Coefficients.
<code>scores</code>	A data frame containing the predictors in the <code>m1m</code> model and the canonical scores on <code>ndim</code> dimensions. These are calculated as <code>Y%%coeffs.raw</code> , where <code>Y</code> contains the standardized response variables.

## Methods (by class)

- `candisc(m1m)`: "m1m" method.

**Methods (by generic)**

- `print(candisc)`: `print()` method for "candisc" objects.
- `summary(candisc)`: `summary()` method for "candisc" objects.
- `coef(candisc)`: `coef()` method for "candisc" objects.
- `plot(candisc)`: "plot" method.

**Author(s)**

Michael Friendly and John Fox

**References**

- Bartlett, M. S. (1938). Further aspects of the theory of multiple regression. Proc. Cambridge Philosophical Society **34**, 33-34.
- Cooley, W.W. & Lohnes, P.R. (1971). Multivariate Data Analysis, New York: Wiley.
- Gittins, R. (1985). Canonical Analysis: A Review with Applications in Ecology, Berlin: Springer.

**See Also**

[candiscList](#), [heplot](#), [heplot3d](#)

**Examples**

```
grass.mod <- lm(cbind(N1,N9,N27,N81,N243) ~ Block + Species, data=Grass)
car::Anova(grass.mod, test="Wilks")

grass.can1 <- candisc(grass.mod, term="Species")
plot(grass.can1)

# library(heplots)
heplot(grass.can1, scale=6, fill=TRUE)

# iris data
iris.mod <- lm(cbind(Petal.Length, Sepal.Length, Petal.Width, Sepal.Width) ~ Species, data=iris)
iris.can <- candisc(iris.mod, data=iris)
#-- assign colors and symbols corresponding to species
col <- c("red", "brown", "green3")
pch <- 1:3
plot(iris.can, col=col, pch=pch)

heplot(iris.can)

# 1-dim plot
iris.can1 <- candisc(iris.mod, data=iris, ndim=1)
plot(iris.can1)
```

---

candiscList	<i>Canonical discriminant analyses</i>
-------------	--

---

### Description

candiscList performs a generalized canonical discriminant analysis for all terms in a multivariate linear model (i.e., an `mlm` object), computing canonical scores and vectors.

### Usage

```
candiscList(mod, ...)

## S3 method for class 'mlm'
candiscList(mod, type = "2", manova, ndim, ...)

## S3 method for class 'candiscList'
print(x, ...)

## S3 method for class 'candiscList'
summary(object, ...)

## S3 method for class 'candiscList'
plot(x, term, ask = interactive(), graphics = TRUE, ...)
```

### Arguments

<code>mod</code>	An <code>mlm</code> object, such as computed by <code>lm()</code> with a multivariate response
<code>...</code>	arguments to be passed down.
<code>type</code>	type of test for the model term, one of: "II", "III", "2", or "3"
<code>manova</code>	the <code>Anova.mlm</code> object corresponding to <code>mod</code> . Normally, this is computed internally by <code>Anova(mod)</code>
<code>ndim</code>	Number of dimensions to store in the means, structure, scores and <code>coeffs.*</code> components. The default is the rank of the H matrix for the hypothesis term.
<code>object, x</code>	A <code>candiscList</code> object
<code>term</code>	The name of one term to be plotted for the <code>plot</code> method. If not specified, one <code>candisc</code> plot is produced for each term in the <code>mlm</code> object.
<code>ask</code>	If TRUE (the default, when running interactively), a menu of terms is presented; if <code>ask</code> is FALSE, canonical plots for all terms are produced.
<code>graphics</code>	if TRUE (the default, when running interactively), then the menu of terms to plot is presented in a dialog box rather than as a text menu.

### Value

An object of class `candiscList` which is a list of "candisc" objects for the terms in the `mlm`.

**Methods (by class)**

- `candiscList(mlm)`: "mlm" method.

**Methods (by generic)**

- `print(candiscList)`: `print()` method for "candiscList" objects.
- `summary(candiscList)`: `summary()` method for "candiscList" objects.
- `plot(candiscList)`: `plot()` method for "candiscList" objects.

**Author(s)**

Michael Friendly and John Fox

**See Also**

[candisc](#), [heplot](#), [heplot3d](#)

**Examples**

```
grass.mod <- lm(cbind(N1,N9,N27,N81,N243) ~ Block + Species, data=Grass)

grass.canL <-candiscList(grass.mod)
names(grass.canL)
names(grass.canL$Species)

## Not run:
print(grass.canL)

## End(Not run)
plot(grass.canL, type="n", ask=FALSE)
heplot(grass.canL$Species, scale=6)
heplot(grass.canL$Block, scale=2)
```

---

can\_lm

*Transform a Multivariate Linear model mlm to a Canonical Representation*

---

**Description**

This function uses [candisc](#) to transform the responses in a multivariate linear model to scores on canonical variables for a given term and then uses those scores as responses in a linear (lm) or multivariate linear model (mlm).

The function constructs a model formula of the form `Can ~ terms` where `Can` is the canonical score(s) and `terms` are the terms in the original mlm, then runs `lm()` with that formula.



**Usage**

```
can_lm(mod, term, ...)
```

**Arguments**

mod	A mlm object
term	One term in that model
...	Arguments passed to <a href="#">candisc</a>

**Value**

A lm object if term is a rank 1 hypothesis, otherwise a mlm object

**Author(s)**

Michael Friendly

**See Also**

[candisc](#), [cancor](#)

**Examples**

```
iris.mod <- lm(cbind(Petal.Length, Sepal.Length, Petal.Width, Sepal.Width) ~ Species, data=iris)
iris.can <- can_lm(iris.mod, "Species")
iris.can
car::Anova(iris.mod)
car::Anova(iris.can)
```

---

dataIndex

*Indices of observations in a model data frame*

---

**Description**

Find sequential indices for observations in a data frame corresponding to the unique combinations of the levels of a given model term from a model object or a data frame

**Usage**

```
dataIndex(x, term)
```

**Arguments**

x	Either a data frame or a model object
term	The name of one term in the model, consisting only of factors

**Value**

A vector of indices.

**Author(s)**

Michael Friendly

**Examples**

```
factors <- expand.grid(A=factor(1:3),B=factor(1:2),C=factor(1:2))
n <- nrow(factors)
responses <- data.frame(Y1=10+round(10*rnorm(n)),Y2=10+round(10*rnorm(n)))

test <- data.frame(factors, responses)
mod <- lm(cbind(Y1,Y2) ~ A*B, data=test)

dataIndex(mod, "A")
dataIndex(mod, "A:B")
```

---

Grass

*Yields from Nitrogen nutrition of grass species*

---

**Description**

The data frame Grass gives the yield (10 \* log10 dry-weight (g)) of eight grass Species in five replicates (Block) grown in sand culture at five levels of nitrogen.

**Format**

A data frame with 40 observations on the following 7 variables.

Species a factor with levels B.media D.glomerata F.ovina F.rubra H.pubesens K.cristata  
L.perenne P.bertolonii

Block a factor with levels 1 2 3 4 5

N1 species yield at 1 ppm Nitrogen

N9 species yield at 9 ppm Nitrogen

N27 species yield at 27 ppm Nitrogen

N81 species yield at 81 ppm Nitrogen

N243 species yield at 243 ppm Nitrogen

## Details

Nitrogen (NaNO<sub>3</sub>) levels were chosen to vary from what was expected to be from critically low to almost toxic. The amount of Nitrogen can be considered on a log<sub>3</sub> scale, with levels 0, 2, 3, 4, 5. Gittins (1985, Ch. 11) treats these as equally spaced for the purpose of testing polynomial trends in Nitrogen level.

The data are also not truly multivariate, but rather a split-plot experimental design. For the purpose of exposition, he regards Species as the experimental unit, so that correlations among the responses refer to a composite representative of a species rather than to an individual exemplar.

## Source

Gittins, R. (1985), *Canonical Analysis: A Review with Applications in Ecology*, Berlin: Springer-Verlag, Table A-5.

## Examples

```
str(Grass)
grass.mod <- lm(cbind(N1,N9,N27,N81,N243) ~ Block + Species, data=Grass)
car::Anova(grass.mod)

grass.canL <-candiscList(grass.mod)
names(grass.canL)
names(grass.canL$Species)
```

---

heplot.cancor

*Canonical Correlation HE plots*

---

## Description

Hypothesis - Error (HE) plots for canonical correlation analysis provide a new graphical method for understanding the relations between two sets of variables, **X** and **Y**. They are similar to HE plots for multivariate multiple regression (MMRA) problems, except that ...

These functions plot ellipses (or ellipsoids in 3D) in canonical space representing the hypothesis and error sums-of-squares-and-products matrices for terms in a multivariate linear model representing the result of a canonical correlation analysis. They provide a low-rank 2D (or 3D) view of the effects in the space of maximum canonical correlations, together with variable vectors representing the correlations of Y variables with the canonical dimensions.

For consistency with [heplot.candisc](#), the plots show effects in the space of the canonical Y variables selected by which.

The interpretation of variable vectors in these plots is different from that of the terms plotted as H "ellipses," which appear as degenerate lines in the plot (because they correspond to 1 df tests of rank(H)=1).

In canonical space, the interpretation of the H ellipses for the terms is the same as in ordinary HE plots: a term is significant *iff* its H ellipse projects outside the (orthogonalized) E ellipsoid

somewhere in the space of the Y canonical dimensions. The orientation of each H ellipse with respect to the Y canonical dimensions indicates which dimensions that X variate contributes to.

On the other hand, the variable vectors shown in these plots are intended only to show the correlations of Y variables with the canonical dimensions. Only their relative lengths and angles with respect to the Y canonical dimensions have meaning. Relative lengths correspond to proportions of variance accounted for in the Y canonical dimensions plotted; angles between the variable vectors and the canonical axes correspond to the structure correlations. The absolute lengths of these vectors are typically manipulated by the scale argument to provide better visual resolution and labeling for the variables.

Setting the aspect ratio of these plots is important for the proper interpretation of angles between the variable vectors and the coordinate axes. However, this then makes it impossible to change the aspect ratio of the plot by re-sizing manually.

### Usage

```
## S3 method for class 'cancor'
heplot(
  mod,
  which = 1:2,
  scale,
  asp = 1,
  var.vectors = "Y",
  var.col = c("blue", "darkgreen"),
  var.lwd = par("lwd"),
  var.cex = par("cex"),
  var.xpd = TRUE,
  prefix = "Ycan",
  suffix = TRUE,
  terms = TRUE,
  ...
)
```

### Arguments

<code>mod</code>	A <code>cancor</code> object
<code>which</code>	A numeric vector containing the indices of the Y canonical dimensions to plot.
<code>scale</code>	Scale factor for the variable vectors in canonical space. If not specified, the function calculates one to make the variable vectors approximately fill the plot window.
<code>asp</code>	aspect ratio setting. Use <code>asp=1</code> in 2D plots and <code>asp="iso"</code> in 3D plots to ensure equal units on the axes. Use <code>asp=NA</code> in 2D plots and <code>asp=NULL</code> in 3D plots to allow separate scaling for the axes. See Details below.
<code>var.vectors</code>	Which variable vectors to plot? A character vector containing one or more of "X" and "Y".
<code>var.col</code>	Color(s) for variable vectors and labels, a vector of length 1 or 2. The first color is used for Y vectors and the second for X vectors, if these are plotted.
<code>var.lwd</code>	Line width for variable vectors

var.cex	Text size for variable vector labels
var.xpd	logical. Allow variable labels outside the plot box? Does not apply to 3D plots.
prefix	Prefix for labels of the Y canonical dimensions.
suffix	Suffix for labels of canonical dimensions. If suffix=TRUE the percent of hypothesis (H) variance accounted for by each canonical dimension is added to the axis label.
terms	Terms for the X variables to be plotted in canonical space. The default, terms=TRUE or terms="X" plots H ellipses for all of the X variables. terms="Xcan" plots H ellipses for all of the X canonical variables, Xcan1, Xcan2, ...
...	Other arguments passed to link[heplots]{heplot}. In particular, you can pass linear hypotheses among the term variables via hypotheses.

**Value**

Returns invisibly an object of class "heplot", with coordinates for the various hypothesis ellipses and the error ellipse, and the limits of the horizontal and vertical axes.

**Author(s)**

Michael Friendly

**References**

Gittins, R. (1985). *Canonical Analysis: A Review with Applications in Ecology*, Berlin: Springer.  
 Mardia, K. V., Kent, J. T. and Bibby, J. M. (1979). *Multivariate Analysis*. London: Academic Press.

**See Also**

[cancor](#) for details on canonical correlation as implemented here; [plot.cancor](#) for scatterplots of canonical variable scores. [heplot.candisc](#), [heplot](#), [linearHypothesis](#)

**Examples**

```
data(Rohwer, package="heplots")
X <- as.matrix(Rohwer[,6:10])
Y <- as.matrix(Rohwer[,3:5])
cc <- cancor(X, Y, set.names=c("PA", "Ability"))

# basic plot
heplot(cc)

# note relationship of joint hypothesis to individual ones
heplot(cc, scale=1.25, hypotheses=list("na+ns"=c("na", "ns")))

# more options
heplot(cc, hypotheses=list("All X"=colnames(X)),
  fill=c(TRUE,FALSE), fill.alpha=0.2,
  var.cex=1.5, var.col="red", var.lwd=3,
  prefix="Y canonical dimension")
```

```

)

# 3D version
## Not run:
heplot3d(cc, var.lwd=3, var.col="red")

## End(Not run)

```

---

heplot.candisc

*Canonical Discriminant HE plots*


---

### Description

These functions plot ellipses (or ellipsoids in 3D) in canonical discriminant space representing the hypothesis and error sums-of-squares-and-products matrices for terms in a multivariate linear model. They provide a low-rank 2D (or 3D) view of the effects for that term in the space of maximum discrimination.

### Usage

```

## S3 method for class 'candisc'
heplot(
  mod,
  which = 1:2,
  scale,
  asp = 1,
  var.col = "blue",
  var.lwd = par("lwd"),
  var.cex = par("cex"),
  var.pos,
  rev.axes = c(FALSE, FALSE),
  prefix = "Can",
  suffix = TRUE,
  terms = mod$term,
  ...
)

```

### Arguments

mod	A candisc object for one term in a mlm
which	A numeric vector containing the indices of the canonical dimensions to plot.
scale	Scale factor for the variable vectors in canonical space. If not specified, the function calculates one to make the variable vectors approximately fill the plot window.

asp	Aspect ratio for the horizontal and vertical dimensions. The defaults, asp=1 for heplot.candisc and asp="iso" for heplot3d.candisc ensure equal units on all axes, so that angles and lengths of variable vectors are interpretable. As well, the standardized canonical scores are uncorrelated, so the Error ellipse (ellipsoid) should plot as a circle (sphere) in canonical space. For heplot3d.candisc, use asp=NULL to suppress this transformation to iso-scaled axes.
var.col	Color for variable vectors and labels
var.lwd	Line width for variable vectors
var.cex	Text size for variable vector labels
var.pos	Position(s) of variable vector labels wrt. the end point. If not specified, the labels are out-justified left and right with respect to the end points.
rev.axes	Logical, a vector of length(which). TRUE causes the orientation of the canonical scores and structure coefficients to be reversed along a given axis.
prefix	Prefix for labels of canonical dimensions.
suffix	Suffix for labels of canonical dimensions. If suffix=TRUE the percent of hypothesis (H) variance accounted for by each canonical dimension is added to the axis label.
terms	Terms from the original mlm whose H ellipses are to be plotted in canonical space. The default is the one term for which the canonical scores were computed. If terms=TRUE, all terms are plotted.
...	Arguments to be passed down to <a href="#">heplot</a> or <a href="#">heplot3d</a>

### Details

The generalized canonical discriminant analysis for one term in a mlm is based on the eigenvalues,  $\lambda_i$ , and eigenvectors,  $V$ , of the H and E matrices for that term. This produces uncorrelated canonical scores which give the maximum univariate F statistics. The canonical HE plot is then just the HE plot of the canonical scores for the given term.

For heplot3d.candisc, the default asp="iso" now gives a geometrically correct plot, but the third dimension, CAN3, is often small. Passing an expanded range in xlim to [heplot3d](#) usually helps.

### Value

heplot.candisc returns invisibly an object of class "heplot", with coordinates for the various hypothesis ellipses and the error ellipse, and the limits of the horizontal and vertical axes.

Similarly, heplot3d.candisc returns an object of class "heplot3d".

### Author(s)

Michael Friendly and John Fox

### References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples *Journal of Statistical Software*, 17(6), 1-42. <https://www.jstatsoft.org/v17/i06/> doi:10.18637/jss.v017.i06

Friendly, M. (2007). HE plots for Multivariate General Linear Models. *Journal of Computational and Graphical Statistics*, **16**(2) 421–444. <http://datavis.ca/papers/jcgs-heplots.pdf>, doi:10.1198/106186007X208407.

### See Also

[candisc](#), [candiscList](#), [heplot](#), [heplot3d](#), [aspect3d](#)

### Examples

```
## Pottery data, from car package
data(Pottery, package = "carData")
pottery.mod <- lm(cbind(Al, Fe, Mg, Ca, Na) ~ Site, data=Pottery)
pottery.can <- candisc(pottery.mod)

heplot(pottery.can, var.lwd=3)
if(requireNamespace("rgl")){
  heplot3d(pottery.can, var.lwd=3, scale=10, zlim=c(-3,3), wire=FALSE)
}

# reduce example for CRAN checks time

grass.mod <- lm(cbind(N1,N9,N27,N81,N243) ~ Block + Species, data=Grass)

grass.can1 <- candisc(grass.mod,term="Species")
grass.canL <- candiscList(grass.mod)

heplot(grass.can1, scale=6)
heplot(grass.can1, scale=6, terms=TRUE)
heplot(grass.canL, terms=TRUE, ask=FALSE)

heplot3d(grass.can1, wire=FALSE)
# compare with non-iso scaling
rgl::aspect3d(x=1,y=1,z=1)
# or,
# heplot3d(grass.can1, asp=NULL)

## Can't run this in example
# rgl::play3d(rgl::spin3d(axis = c(1, 0, 0), rpm = 5), duration=12)

# reduce example for CRAN checks time

## FootHead data, from heplots package
library(heplots)
data(FootHead)

# use Helmert contrasts for group
contrasts(FootHead$group) <- contr.helmert

foot.mod <- lm(cbind(width, circum,front.back,eye.top,ear.top,jaw)~group, data=FootHead)
```



```
foot.can <- candisc(foot.mod)
heplot(foot.can, main="Candisc HE plot",
  hypotheses=list("group.1"="group1", "group.2"="group2"),
  col=c("red", "blue", "green3", "green3" ), var.col="red")
```

---

heplot.candiscList      *Canonical Discriminant HE plots*

---

### Description

These functions plot ellipses (or ellipsoids in 3D) in canonical discriminant space representing the hypothesis and error sums-of-squares-and-products matrices for terms in a multivariate linear model. They provide a low-rank 2D (or 3D) view of the effects for that term in the space of maximum discrimination.

### Usage

```
## S3 method for class 'candiscList'
heplot(mod, term, ask = interactive(), graphics = TRUE, ...)
```

### Arguments

mod	A candiscList object for terms in a mlm
term	The name of one term to be plotted for the heplot and heplot3d methods. If not specified, one plot is produced for each term in the mlm object.
ask	If TRUE (the default), a menu of terms is presented; if ask is FALSE, canonical HE plots for all terms are produced.
graphics	if TRUE (the default, when running interactively), then the menu of terms to plot is presented in a dialog box rather than as a text menu.
...	Arguments to be passed down

### Value

No useful value; used for the side-effect of producing canonical HE plots.

### Author(s)

Michael Friendly and John Fox

## References

Friendly, M. (2006). Data Ellipses, HE Plots and Reduced-Rank Displays for Multivariate Linear Models: SAS Software and Examples *Journal of Statistical Software*, 17(6), 1-42. <https://www.jstatsoft.org/v17/i06/> doi:10.18637/jss.v017.i06.

Friendly, M. (2007). HE plots for Multivariate General Linear Models. *Journal of Computational and Graphical Statistics*, 16(2) 421–444. <http://datavis.ca/papers/jcgs-heplots.pdf>, doi:10.1198/106186007X208407.

## See Also

[candisc](#), [candiscList](#), [heplot](#), [heplot3d](#)

---

HSB

*High School and Beyond Data*

---

## Description

The High School and Beyond Project was a longitudinal study of students in the U.S. carried out in 1980 by the National Center for Education Statistics. Data were collected from 58,270 high school students (28,240 seniors and 30,030 sophomores) and 1,015 secondary schools. The HSB data frame is sample of 600 observations, of unknown characteristics, originally taken from Tatsuoka (1988).

## Format

A data frame with 600 observations on the following 15 variables. There is no missing data.

id Observation id: a numeric vector

gender a factor with levels male female

race Race or ethnicity: a factor with levels hispanic asian african-amer white

ses Socioeconomic status: a factor with levels low middle high

sch School type: a factor with levels public private

prog High school program: a factor with levels general academic vocation

locus Locus of control: a numeric vector

concept Self-concept: a numeric vector

mot Motivation: a numeric vector

career Career plan: a factor with levels clerical craftsman farmer homemaker laborer manager  
military operative prof1 prof2 proprietor protective sales school service technical  
not working

read Standardized reading score: a numeric vector

write Standardized writing score: a numeric vector

math Standardized math score: a numeric vector

sci Standardized science score: a numeric vector

ss Standardized social science (civics) score: a numeric vector

**Source**

Tatsuoka, M. M. (1988). *Multivariate Analysis: Techniques for Educational and Psychological Research* (2nd ed.). New York: Macmillan, Appendix F, 430-442.

**References**

High School and Beyond data files: <http://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/7896>

**Examples**

```
str(HSB)
# main effects model
hsb.mod <- lm( cbind(read, write, math, sci, ss) ~
  gender + race + ses + sch + prog, data=HSB)
car::Anova(hsb.mod)

# Add some interactions
hsb.mod1 <- update(hsb.mod, . ~ . + gender:race + ses:prog)
heplot(hsb.mod1, col=palette()[c(2,1,3:6)], variables=c("read","math"))

hsb.can1 <- candisc(hsb.mod1, term="race")
heplot(hsb.can1, col=c("red", "black"))

# show canonical results for all terms
## Not run:
hsb.can <- candiscList(hsb.mod)
hsb.can

## End(Not run)
```

---

plot.cancor

*Canonical Correlation Plots*


---

**Description**

This function produces plots to help visualize X, Y data in canonical space.

The present implementation plots the canonical scores for the Y variables against those for the X variables on given dimensions. We treat this as a view of the data in canonical space, and so offer additional annotations to a standard scatterplot.

Canonical correlation analysis assumes that the all correlations between the X and Y variables can be expressed in terms of correlations the canonical variate pairs, (Xcan1, Ycan1), (Xcan2, Ycan2), ..., and that the relations between these pairs are indeed linear.

Data ellipses, and smoothed (loess) curves, together with the linear regression line for each canonical dimension help to assess whether there are peculiarities in the data that might threaten the validity of CCA. Point identification methods can be useful to determine influential cases.

**Usage**

```
## S3 method for class 'cancor'
plot(
  x,
  which = 1,
  xlim,
  ylim,
  xlab,
  ylab,
  points = TRUE,
  add = FALSE,
  col = palette()[1],
  ellipse = TRUE,
  ellipse.args = list(),
  smooth = FALSE,
  smoother.args = list(),
  col.smooth = palette()[3],
  abline = TRUE,
  col.lines = palette()[2],
  lwd = 2,
  labels = rownames(xy),
  id.method = "mahal",
  id.n = 0,
  id.cex = 1,
  id.col = palette()[1],
  ...
)
```

**Arguments**

x	A "cancor" object
which	Which dimension to plot? An integer in 1:x\$ndim.
xlim, ylim	Limits for x and y axes
xlab, ylab	Labels for x and y axes. If not specified, these are constructed from the set.names component of x.
points	logical. Display the points?
add	logical. Add to an existing plot?
col	Color for points.
ellipse	logical. Draw a data ellipse for the canonical scores?
ellipse.args	A list of arguments passed to <a href="#">dataEllipse</a> . Internally, the function sets the default value for levels to 0.68.
smooth	logical. Draw a (loess) smoothed curve?
smoother.args	Arguments passed to <a href="#">loessLine</a> , which should be consulted for details and defaults.
col.smooth	Color for the smoothed curve.

abline	logical. Draw the linear regression line for Ycan[,which] on Xcan[,which]?
col.lines	Color for the linear regression line
lwd	Line widths
labels	Point labels for point identification via the id.method argument.
id.method	Method used to identify individual points. See <a href="#">showLabels</a> for details. The default, id.method = "mahal" identifies the id.n points furthest from the centroid.
id.n	Number of points to identify
id.cex, id.col	Character size and color for labeled points
...	Other arguments passed down to plot(...) and points(...)

**Value**

None. Used for its side effect of producing a plot. the value returned

**Author(s)**

Michael Friendly

**References**

Mardia, K. V., Kent, J. T. and Bibby, J. M. (1979). *Multivariate Analysis*. London: Academic Press.

**See Also**

[cancor](#),  
[dataEllipse](#), [loessLine](#), [showLabels](#)

**Examples**

```
data(Rohwer, package="heplots")
X <- as.matrix(Rohwer[,6:10]) # the PA tests
Y <- as.matrix(Rohwer[,3:5]) # the aptitude/ability variables

cc <- cancor(X, Y, set.names=c("PA", "Ability"))

plot(cc)
# exercise some options
plot(cc, which=1,
      smooth=TRUE,
      pch = 16,
      id.n=3, ellipse.args=list(fill=TRUE, fill.alpha = 0.2))
plot(cc, which=2, smooth=TRUE)
plot(cc, which=3, smooth=TRUE)

# plot vectors showing structure correlations of Xcan and Ycan with their own variables
plot(cc)
struc <- cc$structure
```

```

Xstruc <- struc$X.scores[,1]
Ystruc <- struc$Y.scores[,1]
scale <- 2

# place vectors in the margins of the plot
usr <- matrix(par("usr"), nrow=2, dimnames=list(c("min", "max"), c("x", "y")))
ypos <- usr[2,2] - (1:5)/10
arrows(0, ypos, scale*Xstruc, ypos, angle=10, len=0.1, col="blue")
text(scale*Xstruc, ypos, names(Xstruc), pos=2, col="blue")

xpos <- usr[2,1] - ( 1 + 1:3)/10
arrows(xpos, 0, xpos, scale*Ystruc, angle=10, len=0.1, col="darkgreen")
text(xpos, scale*Ystruc, names(Ystruc), pos=1, col="darkgreen")

```

---

predictor.names	<i>Get predictor names from a lm-like model</i>
-----------------	---

---

## Description

Get predictor names from a lm-like model

## Usage

```
predictor.names(model, ...)
```

```
## Default S3 method:
predictor.names(model, ...)
```

## Arguments

model	Model object
...	other arguments (ignored)

## Value

A character vector of variable names

## Methods (by class)

- predictor.names(default): "default" method.

## Examples

```
#none
```

---

redundancy                      *Canonical Redundancy Analysis*

---

### Description

Calculates indices of redundancy (Stewart & Love, 1968) from a canonical correlation analysis. These give the proportion of variances of the variables in each set (X and Y) which are accounted for by the variables in the other set through the canonical variates.

### Usage

```
redundancy(object, ...)
```

```
## S3 method for class 'cancor.redundancy'
print(x, digits = max(getOption("digits") - 3, 3), ...)
```

### Arguments

object	A "cancor" object
...	Other arguments
x	A "cancor.redundancy" for the print method.
digits	Number of digits to print

### Details

The term "redundancy analysis" has a different interpretation and implementation in the environmental ecology literature, such as the **vegan**. In that context, each  $Y_i$  variable is regressed separately on the predictors in  $X$ , to give fitted values  $\hat{Y} = [\hat{Y}_1, \hat{Y}_2, \dots]$ . Then a PCA of  $\hat{Y}$  is carried out to determine a reduced-rank structure of the predictions.

### Value

An object of class "cancor.redundancy", a list with the following 5 components:

Xcan.redun	Canonical redundancies for the X variables, i.e., the total fraction of X variance accounted for by the Y variables through each canonical variate.
Ycan.redun	Canonical redundancies for the Y variables
X.redun	Total canonical redundancy for the X variables, i.e., the sum of Xcan.redun.
Y.redun	Total canonical redundancy for the Y variables
set.names	names for the X and Y sets of variables

### Functions

- `print(cancor.redundancy)`: `print()` method for "cancor.redundancy" objects.

**Author(s)**

Michael Friendly

**References**

Muller K. E. (1981). Relationships between redundancy analysis, canonical correlation, and multivariate regression. *Psychometrika*, **46**(2), 139-42.

Stewart, D. and Love, W. (1968). A general canonical correlation index. *Psychological Bulletin*, **70**, 160-163.

Brainder, "Redundancy in canonical correlation analysis", <https://brainder.org/2019/12/27/redundancy-in-canonical-correlation-analysis/>

**See Also**

[cancor](#)

**Examples**

```
data(Rohwer, package="heplots")
X <- as.matrix(Rohwer[,6:10]) # the PA tests
Y <- as.matrix(Rohwer[,3:5])  # the aptitude/ability variables

cc <- cancor(X, Y, set.names=c("PA", "Ability"))

redundancy(cc)
##
## Redundancies for the PA variables & total X canonical redundancy
##
##      Xcan1      Xcan2      Xcan3 total X|Y
##  0.17342  0.04211  0.00797  0.22350
##
## Redundancies for the Ability variables & total Y canonical redundancy
##
##      Ycan1      Ycan2      Ycan3 total Y|X
##  0.2249  0.0369  0.0156  0.2774
```

---

varOrder

*Order variables according to canonical structure or other criteria*


---

**Description**

The varOrder function implements some features of “effect ordering” (Friendly & Kwan (2003) for *variables* in a multivariate data display to make the displayed relationships more coherent.

This can be used in pairwise HE plots, scatterplot matrices, parallel coordinate plots, plots of multivariate means, and so forth.



For a numeric data frame, the most useful displays often order variables according to the angles of variable vectors in a 2D principal component analysis or biplot. For a multivariate linear model, the analog is to use the angles of the variable vectors in a 2D canonical discriminant biplot.

### Usage

```
varOrder(x, ...)

## S3 method for class 'mlm'
varOrder(
  x,
  term,
  variables,
  type = c("can", "pc"),
  method = c("angles", "dim1", "dim2", "alphabet", "data", "colmean"),
  names = FALSE,
  descending = FALSE,
  ...
)

## S3 method for class 'data.frame'
varOrder(
  x,
  variables,
  method = c("angles", "dim1", "dim2", "alphabet", "data", "colmean"),
  names = FALSE,
  descending = FALSE,
  ...
)

## Default S3 method:
varOrder(x, ...)
```

### Arguments

x	A multivariate linear model or a numeric data frame
...	Arguments passed to methods
term	For the <code>mlm</code> method, one term in the model for which the canonical structure coefficients are found.
variables	indices or names of the variables to be ordered; defaults to all response variables an MLM or all numeric variables in a data frame.
type	For an MLM, <code>type="can"</code> uses the canonical structure coefficients for the given term; <code>type="pc"</code> uses the principal component variable eigenvectors.
method	One of <code>c("angles", "dim1", "dim2", "alphabet", "data", "colmean")</code> giving the effect ordering method. <b>"angles"</b> Orders variables according to the angles their vectors make with dimensions 1 and 2, counter-clockwise starting from the lower-left quadrant in a 2D biplot or candisc display.

	<b>"dim1"</b>	Orders variables in increasing order of their coordinates on dimension 1
	<b>"dim2"</b>	Orders variables in increasing order of their coordinates on dimension 2
	<b>"alphabet"</b>	Orders variables alphabetically
	<b>"data"</b>	Uses the order of the variables in the data frame or the list of responses in the MLM
	<b>"colmean"</b>	Uses the order of the column means of the variables in the data frame or the list of responses in the MLM
names		logical; if TRUE the effect ordered names of the variables are returned; otherwise, their indices in variables are returned.
descending		If TRUE, the ordered result is reversed to a descending order.

**Value**

A vector of integer indices of the variables or a character vector of their names.

**Methods (by class)**

- varOrder(mlm): "mlm" method.
- varOrder(data.frame): "data.frame" method.
- varOrder(default): "default" method.

**Author(s)**

Michael Friendly

**References**

Friendly, M. & Kwan, E. (2003). Effect Ordering for Data Displays, *Computational Statistics and Data Analysis*, **43**, 509-539. doi:[10.1016/S01679473\(02\)002906](https://doi.org/10.1016/S01679473(02)002906)

**Examples**

```
data(Wine, package="candisc")
Wine.mod <- lm(as.matrix(Wine[, -1]) ~ Cultivar, data=Wine)
Wine.can <- candisc(Wine.mod)
plot(Wine.can, ellipse=TRUE)

# pairs.mlm HE plot, variables in given order
pairs(Wine.mod, fill=TRUE, fill.alpha=.1, var.cex=1.5)

order <- varOrder(Wine.mod)
pairs(Wine.mod, variables=order, fill=TRUE, fill.alpha=.1, var.cex=1.5)
```

---

vecscale *Scale vectors to fill the current plot*

---

### Description

Calculates a scale factor so that a collection of vectors nearly fills the current plot, that is, the longest vector does not extend beyond the plot region.

### Usage

```
vecscale(  
  vectors,  
  bbox = matrix(par("usr"), 2, 2),  
  origin = c(0, 0),  
  factor = 0.95  
)
```

### Arguments

vectors	a two-column matrix giving the end points of a collection of vectors
bbox	the bounding box of the containing plot region within which the vectors are to be plotted
origin	origin of the vectors
factor	maximum length of the rescaled vectors relative to the maximum possible

### Value

scale factor, the multiplier of the vectors

### Author(s)

Michael Friendly

### See Also

[vectors](#)

### Examples

```
bbox <- matrix(c(-3, 3, -2, 2), 2, 2)  
colnames(bbox) <- c("x", "y")  
rownames(bbox) <- c("min", "max")  
bbox  
  
vecs <- matrix( runif(10, -1, 1), 5, 2)  
  
plot(bbox)  
arrows(0, 0, vecs[,1], vecs[,2], angle=10, col="red")
```

```
(s <- vecscale(vecs))
arrows(0, 0, s*vecs[,1], s*vecs[,2], angle=10)
```

---

vectors

*Draw Labeled Vectors in 2D or 3D*


---

### Description

Graphics utility functions to draw vectors from an origin to a collection of points (using `arrows` in 2D or `lines3d` in 3D) with labels for each (using `text` or `texts3d`).

### Usage

```
vectors(
  x,
  origin = c(0, 0),
  labels = rownames(x),
  scale = 1,
  col = "blue",
  lwd = 1,
  cex = 1,
  length = 0.1,
  angle = 13,
  pos = NULL,
  ...
)
```

### Arguments

<code>x</code>	A two-column matrix or a three-column matrix containing the end points of the vectors
<code>origin</code>	Starting point(s) for the vectors
<code>labels</code>	Labels for the vectors
<code>scale</code>	A multiplier for the length of each vector
<code>col</code>	color(s) for the vectors.
<code>lwd</code>	line width(s) for the vectors.
<code>cex</code>	color(s) for the vectors.
<code>length</code>	For vectors, length of the edges of the arrow head (in inches).
<code>angle</code>	For vectors, angle from the shaft of the arrow to the edge of the arrow head.
<code>pos</code>	For vectors, position of the text label relative to the vector head. If <code>pos==NULL</code> , labels are positioned labels outside, relative to arrow ends.
<code>...</code>	other graphical parameters, such as <code>lty</code> , <code>xpd</code> , ...

**Details**

The graphical parameters `col`, `lty` and `lwd` can be vectors of length greater than one and will be recycled if necessary

**Value**

None

**Author(s)**

Michael Friendly

**See Also**

[arrows](#), [text](#), [segments](#)

[lines3d](#), [texts3d](#)

**Examples**

```
plot(c(-3, 3), c(-3,3), type="n")
X <- matrix(rnorm(10), ncol=2)
rownames(X) <- LETTERS[1:5]
vectors(X, scale=2, col=palette())
```

---

Wilks

*Wilks Lambda Tests for Canonical Correlations*

---

**Description**

Tests the sequential hypotheses that the  $i$ th canonical correlation and all that follow it are zero,

$$\rho_i = \rho_{i+1} = \dots = 0$$

**Usage**

```
Wilks(object, ...)
```

```
## S3 method for class 'cancel'
```

```
Wilks(object, ...)
```

```
## S3 method for class 'candisc'
```

```
Wilks(object, ...)
```

**Arguments**

`object` An object of class "cancel" or `\code{"candisc"}`

`...` Other arguments passed to methods (not used)

**Details**

Wilks' Lambda values are calculated from the eigenvalues and converted to F statistics using Rao's approximation.

**Value**

A data.frame (of class "anova") containing the test statistics

**Methods (by class)**

- Wilks(cancor): "cancor" method.
- Wilks(candisc): print() method for "candisc" objects.

**Author(s)**

Michael Friendly

**References**

Mardia, K. V., Kent, J. T. and Bibby, J. M. (1979). *Multivariate Analysis*. London: Academic Press.

**See Also**

[cancor](#), [~~~~](#)

**Examples**

```
data(Rohwer, package="heplots")
X <- as.matrix(Rohwer[,6:10]) # the PA tests
Y <- as.matrix(Rohwer[,3:5])  # the aptitude/ability variables

cc <- cancor(X, Y, set.names=c("PA", "Ability"))
Wilks(cc)

iris.mod <- lm(cbind(Petal.Length, Sepal.Length, Petal.Width, Sepal.Width) ~ Species, data=iris)
iris.can <- candisc(iris.mod, data=iris)
Wilks(iris.can)
```

---

Wine

*Chemical composition of three cultivars of wine*

---

**Description**

These data are the results of a chemical analysis of wines grown in the same region in Italy but derived from three different cultivars. The analysis determined the quantities of 13 constituents found in each of the three types of wines.

**Format**

A data frame with 178 observations on the following 14 variables.

Cultivar a factor with levels barolo grignolino barbera

Alcohol a numeric vector

MalicAcid a numeric vector

Ash a numeric vector

AlcAsh a numeric vector, Alkalinity of ash

Mg a numeric vector, Magnesium

Phenols a numeric vector, Total phenols

Flav a numeric vector, Flavanoids

NonFlavPhenols a numeric vector

Proa a numeric vector, Proanthocyanins

Color a numeric vector, color intensity

Hue a numeric vector

OD a numeric vector, OD280/OD315 of diluted wines

Proline a numeric vector

**Details**

This data set is a classic in the machine learning literature as an easy high-D classification problem, but is also of interest for examples of MANOVA and discriminant analysis.

The precise definitions of these variables is unknown: units, how they were measured, etc.

**Source**

This data set was obtained from the UCI Machine Learning Repository, <http://archive.ics.uci.edu/ml/datasets/Wine>

This page references a large number of papers that use this data set to compare different methods.

**References**

In R, a comparable data set is contained in the **ggbiplot** package.

**Examples**

```
data(Wine)
str(Wine)
#summary(Wine)

Wine.mlm <- lm(as.matrix(Wine[, -1]) ~ Cultivar, data=Wine)
Wine.can <- candisc(Wine.mlm)
Wine.can

plot(Wine.can, ellipse=TRUE)
plot(Wine.can, which=1)
```

---

Wolves

*Wolf skulls*


---

### Description

Skull morphometric data on Rocky Mountain and Arctic wolves (*Canis Lupus L.*) taken from Morrison (1990), originally from Jolicoeur (1959).

### Format

A data frame with 25 observations on the following 11 variables.

group a factor with levels ar:f ar:m rm:f rm:m, comprising the combinations of location and sex

location a factor with levels ar=Arctic, rm=Rocky Mountain

sex a factor with levels f=female, m=male

x1 palatal length, a numeric vector

x2 postpalatal length, a numeric vector

x3 zygomatic width, a numeric vector

x4 palatal width outside first upper molars, a numeric vector

x5 palatal width inside second upper molars, a numeric vector

x6 postglenoid foramina width, a numeric vector

x7 interorbital width, a numeric vector

x8 braincase width, a numeric vector

x9 crown length, a numeric vector

### Details

All variables are expressed in millimeters.

The goal was to determine how geographic and sex differences among the wolf populations are determined by these skull measurements. For MANOVA or (canonical) discriminant analysis, the factors group or location and sex provide alternative parameterizations.

### Source

Morrison, D. F. *Multivariate Statistical Methods*, (3rd ed.), 1990. New York: McGraw-Hill, p. 288-289.

### References

Jolicoeur, P. "Multivariate geographical variation in the wolf *Canis lupis L.*", *Evolution*, XIII, 283–299.



**Examples**

```
data(Wolves)

# using group
wolf.mod <-lm(cbind(x1,x2,x3,x4,x5,x6,x7,x8,x9) ~ group, data=Wolves)
car::Anova(wolf.mod)

wolf.can <-candisc(wolf.mod)
plot(wolf.can)
heplot(wolf.can)

# using location, sex
wolf.mod2 <-lm(cbind(x1,x2,x3,x4,x5,x6,x7,x8,x9) ~ location*sex, data=Wolves)
car::Anova(wolf.mod2)

wolf.can2 <-candiscList(wolf.mod2)
plot(wolf.can2)
```

# Index

- \* **MANOVA**
  - Grass, 18
  - Wine, 38
- \* **MMRA**
  - HSB, 26
- \* **aplot**
  - vectors, 36
- \* **cancor**
  - HSB, 26
- \* **candisc**
  - Grass, 18
  - Wine, 38
- \* **datasets**
  - Grass, 18
  - HSB, 26
  - Wine, 38
  - Wolves, 40
- \* **hplot**
  - candisc, 10
  - candiscList, 15
  - heplot.cancor, 19
  - heplot.candisc, 22
  - heplot.candiscList, 25
  - plot.cancor, 27
- \* **hstest**
  - Wilks, 37
- \* **manip**
  - dataIndex, 17
  - varOrder, 32
  - vecscale, 35
- \* **multivariate**
  - cancor, 4
  - candisc, 10
  - candisc-package, 2
  - candiscList, 15
  - heplot.cancor, 19
  - heplot.candisc, 22
  - heplot.candiscList, 25
  - redundancy, 31
  - varOrder, 32
- \* **package**
  - candisc-package, 2
- \* **utilities**
  - dataIndex, 17
- arrows, 36, 37
- aspect3d, 24
- can\_lm, 16
- cancor, 3, 4, 4, 8, 17, 21, 29, 32, 38
- candisc, 4, 8, 10, 16, 17, 24, 26
- candisc-package, 2
- candiscList, 14, 15, 24, 26
- cc, 8
- cca, 8
- coef.cancor (cancor), 4
- coef.candisc (candisc), 10
- cov.wt, 5
- dataEllipse, 28, 29
- dataIndex, 17
- Grass, 18
- heplot, 4, 14, 16, 21, 23, 24, 26
- heplot.cancor, 3, 6, 8, 19
- heplot.candisc, 3, 19, 21, 22
- heplot.candiscList, 25
- heplot3d, 14, 16, 23, 24, 26
- heplot3d.cancor, 3
- heplot3d.cancor (heplot.cancor), 19
- heplot3d.candisc, 3
- heplot3d.candisc (heplot.candisc), 22
- heplot3d.candiscList (heplot.candiscList), 25
- heplots, 4, 8
- HSB, 26
- linearHypothesis, 21
- lines3d, 36, 37

loessLine, 28, 29

palette, 11

plot.cancor, 3, 8, 21, 27

plot.candisc, 3

plot.candisc (candisc), 10

plot.candiscList (candiscList), 15

predictor.names, 30

print.cancor (cancor), 4

print.cancor.redundancy (redundancy), 31

print.candisc (candisc), 10

print.candiscList (candiscList), 15

redundancy, 8, 31

scores (cancor), 4

segments, 37

showLabels, 29

summary.cancor (cancor), 4

summary.candisc (candisc), 10

summary.candiscList (candiscList), 15

text, 36, 37

texts3d, 36, 37

varOrder, 3, 32

vecscale, 35

vectors, 35, 36

vectors3d (vectors), 36

Wilks, 37

Wine, 38

Wolves, 40